SPECIFIC WAYS OF WORKING

CHAPTER 3

Collaborative Work

The following extract from the National Curriculum Council’s *Non-Statutory Guidance for Mathematics* shows that the importance of collaborative work is being recognised:

‘5.8 Activities should where appropriate, involve both independent and co-operative work

* . . . the adult world involves much work in groups and teams of various sizes and types. Pupils should be encouraged to undertake work in teams, discussing mathematical problems, evaluating ideas and alternative solutions, and jointly finding ways to “crack” an onerous assignment.’

*page B9, Non-Statutory Guidance*

A good way to move towards co-operative work is to organise your students in pairs. There are many tasks, particularly practical and problem-solving tasks, where the students will see that working in a pair is a distinct advantage. For instance, in a practical measurement activity involving the use of a steel rule, two people can manage the physical process of measuring, and writing the result down, better than one person on her own.

**Exercise**

*Activity*¹ for a teacher and her class

*Aim of task:* For students to improve their ability to estimate time.

Each pair will need: —a timepiece which measures seconds;  
—paper and something to write with.

One of the pair keeps her eyes closed for an estimated minute, and the other times her. They take it in turns, and keep a record of their results. Some students might like to draw an improvement graph.

¹ Adapted from a suggestion at the SMILE Conference 1989. Thank you to whoever the idea originated from.
As well as the sharing of aspects of a task, a major advantage of group work is the mathematical discussion it generates. It is clear from the many references to oral work in the National Curriculum that the importance of mathematical discussion is becoming increasingly recognised. Students need to practice expressing themselves precisely. There are many tasks which lend themselves to mathematical communication. Students who have shared the work of generating results for a lengthy investigation, for instance, will then have to communicate their results to one another. Other tasks can be designed specifically to facilitate discussion. The following exercise is an example:

**Exercise**

*Activity*¹ for a teacher and her class

Each *pair* will need a supply of centicubes.

One of the pair builds a shape from centicubes, making sure that the partner cannot see it.

She then describes it as precisely as possible to the partner, who tries to replicate it.

At the end of this process the two shapes are compared. Usually they will be different. The pair discuss why they are different.

The pair swap roles.

As pairs develop expertise, they may build more complex shapes.

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¹ Adapted from a suggestion at a SMILE Conference. Thank you to whoever the idea originated from.

**Exercise**

Use the activity *Estimating and Measuring Length* (p.88) with a group class of students.

The mathematical discussion and the sharing of the work required for a task are the two major justifications for working in groups of two. However, in a large group other advantages became apparent. A larger group can generally make a more workable plan for executing a lengthy problem than an individual.

Individual students will have different strengths; the group will have all those strengths. This is why the finished product of a group working cooperatively is likely to be of a higher standard than any of the individual members of the group could have achieved alone.
Chapter 3: Specific Ways of Working, continued

Exercise

Activity for a teacher and her class

Choose a suitable class with whom you could use Loopy Numbers (p.94), Flow-Chart (p.90) or Pinboard Paths (p.103).

Organise the class to work in threes or fours.

Tell them you want a group effort. In both these activities students in the group can share out starting numbers, saving duplication of effort and generating results more quickly. They will then share results and discuss them. Improved techniques for generating results will also be shared and discussed.

Each student could produce an individual account of the conclusions.

You should observe that the conclusions from the investigations above are better than you would have expected from individuals working alone. The exercise above can be completed in one lesson. The advantages of team work become even more apparent in more ‘product’ oriented problem-solving tasks which could take several weeks.

When you have had some practice with this way of working, look more closely at what goes on when a group is planning something together, or having a discussion.

Exercise

Use the following checklist in a collaborative lesson.

Can you see instances of individuals:
— making suggestions?
— taking notice of other’s suggestions?
— providing justifications?
— trying to convince others?
— pausing for thought?
— listening?
— supporting and encouraging others?
— directing the group to the task in hand?
— drawing attention to information?
Chapter 3: Specific Ways of Working, continued

The first page of work on *Pinboard Paths* from one of a pair of students.

There are lots

There are 5 in a square.

There are at least 20 on the big square. Because there is 5 in each square.

I can't find them all.
I am going to do all the ones that start 1.

My friend is going to do the ones that start 2.
Extended Work

Extended work is for everyone. There has been a tendency in the past to regard extended work as something which is only suitable for high-attaining students. This is not the case. Provided a topic is found which can provide sufficient motivation, every student can be involved in extended work. In fact GCSE requires that every student must be involved in extended work. It is also part of the National Criteria for mathematics GCSE. (See 3.17 Appendix B)

The spirit of this is carried through into the National Curriculum:

'Activities should be balanced between those which are short in duration and those which have scope for development over an extended period.'

Different personal qualities are developed by tackling tasks of varying length and complexity. It is useful for pupils to experience the satisfaction both of instant success and of the rewards gained from the in-depth study of extended pieces of work.'

The best kind of extended work always follows students' own interests. The students will be better motivated and therefore achieve more. Tasks imposed by teachers or examination boards could be unfair, as some students may not like the choice of topic. There should at least be a choice for the student.

Activities which follow on from something which a student has already done, and enjoyed, provide a good basis for extended work. An investigation or practical problem can be extended, for instance, to involve the collection or generation of more data and the refinement of conclusions. Build on the students' enthusiasm. If they have already 'completed' a task to their level of ability they could be encouraged to extend it 'sideways' i.e. to widen the scope of the task.

Examples

Where a task has involved surveying their own class, extend it to a survey of another class. Students are likely to improve their survey techniques in the process.

Where a task has involved investigating a certain process using small whole numbers, repeat the process with very large whole numbers.

Sometimes students will work on a short task without bringing the best of their mathematics to bear on it. This could be, for instance, because there was not enough time. A student capable of producing a reasoned argument to justify a particular generalisation in an investigation may not have had time to reach the generalisation. In this case, there is good reason for extending the task for as long as the student needs to complete it in her own terms.

An extended task could be as short as three lessons or it could go on for half a term. For some students, who have difficulty sustaining an activity for 10 minutes, a task lasting a whole lesson could count as extended work. Lower down the school, you can begin by introducing extended tasks of about three lessons or a week's worth of mathematics lessons.
Examples continued

These periods of time can be lengthened as students proceed through the school, so that by the time they reach year 10 they can produce extended pieces of work of the length required for GCSE. Standard Assessment Tasks for the National Curriculum for Key Stage 3 are not likely to take more than one week of maths lessons each. Most will be considerably shorter.

There are different ways of organising extended work in the classroom. The most usual way is for the teacher to set aside a period of time when the whole class will work on their extended pieces of work. An alternative way is to give students the opportunity to pursue their extended tasks at a different time from the rest of the class.

Whether the whole class are doing extended work at the same time or at different times there will need to be some sort of introduction from the teacher.

The way the teacher introduces the project to the students is very important, as it sets the context for the students activities over the duration of the project. It is particularly important, where GCSE or National Curriculum Assessments are involved, to set the parameters clearly.

The following pages guide you through three lessons in which groups of students complete an extended task. We are thinking of three ‘double’ lessons of an hour or more each. If your lessons are shorter than this, adjust the number of lessons accordingly. The extended work indicated is collaborative problem-solving.

Preparation

Read through all that follows before you begin the extended work with students. The italic passages indicate what the students will be doing. The rest is advice and guidance for the teacher.

Choose some topics or tasks that you feel could be completed successfully in three lessons, e.g. Sport Rules or the extension on Star Signs from Section Three. It is best to choose topics which focus on the students’ own interests. Here are some suggested topics:

- Bikes
- Planning a journey
- Designing a game
- Musical tastes

- Clothes
- Fitness
- Food
- Films

See also the list of ideas for extended work at the end of this chapter.

Each of these topics is very broad and needs much more precise definition if it is to produce a task which can be completed in the time available. This is best done by the students.

If students have their own ideas to follow up, that is even better. If not, select about half a dozen different topics or tasks which will provide variety in terms of content as well as style. Choose questions which vary in their degree of open-ness, i.e. choose some, like Sport Rules or Star Signs, where the question is already defined, and some like Bikes or Films, where the students have to define their own question.
LESSON 1

Arrange the class in pairs or small groups.

Tell the students the three-lesson plan in your own words. Tell them your selected list of topics or tasks.
Ask each group to choose a topic or task to work on.
Tell the students that they will be making their own plans about how to proceed with the activity and that the first thing they must do is make sure they have a question to pursue. For some topics, this could take quite a lot of the group's time at the beginning. Problem-posing is important and should not be rushed.

The groups begin by discussing the problem freely. Gradually they begin to focus their attention on some particular aspect. As the lesson proceeds, the groups decide precisely how they are going to pursue the problem. Their plans should indicate the roles of the individuals within the group. They write their plans down. They make a list of any resources they will need. They arrange amongst themselves, and with you, for these resources to be made available next lesson.

Make sure you get a chance to listen to each group for long enough to be clear about what they are doing. It is often better just to listen rather than to interrupt the group's discussion.

By the end of the lesson, make sure every group knows what they will be doing next lesson, what they need to bring, and what you are bringing for them.

If many of the groups are planning to do surveys or questionnaires using other students in the class, encourage some to go out to other classes instead. In this way, the working of groups is not continuously interrupted by other groups asking questions or expecting them to fill in questionnaires.

BETWEEN LESSONS 1 AND 2

There will not be any written work for you to mark after this lesson, but you will probably have some homework—collecting resources or organising things with other teachers.

You can use the free time to familiarise yourself with the students' plans. You could speculate about what they might actually be doing next lesson.

LESSON 2

Do not force the students to follow their plans to the letter. Make the resources which you were asked to bring available for students to take or leave as a normal part of the classroom provision. This gives them the opportunity to select from the resources and possibly to alter their plans as they go along.

The groups execute their plans. At times they will appear to be working as individuals. If they have shared a large task between them, this is quite natural. Some students may leave the classroom to get information, to use or fetch resources, or to survey other classes. They may have to adapt their plans as they go along. This could be, for instance, because of lack of availability of some item of information, or because they may realise that they have begun on a project which is too big for them. The group should discuss and share their work before the lesson ends. They should begin to think about presentation.
LESSON 2 continued

During this second lesson, groups may work quite independently. Again, make sure that you visit each group. Ask them to explain to you what they are doing. Towards the end of the lesson, encourage groups to finish research or trials. This could be surveys, or collecting information from reference material, newspapers or magazines, or testing out a prototype.

LESSON 3

In the final lesson, have plenty of display resources available for the final products:

- Poster paper of various colours
- Coloured pens, thick and thin
- Graph paper and squared paper
- Resources requested by students, etc.

The groups present their results. These could be, for instance, a solution to a problem, or a display of data with conclusions drawn from it. Each individual’s contribution to the presentation should be unique; each may have drawn different conclusions from the same data but the group does not need to come to agreement; both sets of conclusions can be included in the final presentation. A folder of work could be produced and, again, each sheet should be different. It should never include duplicated work in different handwriting.

Methods of presentation:

- Writing of presentation:
- Drawing illustrations
- Drawing bar graphs
- Producing a poster
- Collecting papers together into a folder

Organise a time, either at the end of this lesson or later, when the groups can view one another’s work. The class might like to consider an exhibition for other classes in the school.

In these lessons there is plenty of opportunity for discussion, because the students are working with their peers. Even in individual Extended Tasks, make sure that students have the opportunity to discuss their plans with one another.

The examples given are extended problem-solving activities. Extended investigations will not follow the same path; there is less need for planning. Students can follow their intuition and data is generated rather than collected. Each conclusion can lead to a justification or further question.

The lessons described are based on the most usual way of organising extended work, where all the students are doing their extended work at the same time.

It is administratively easier to tackle it this way to begin with. However, as students get more used to extended work, they themselves will begin to see the potential in other classroom work they are doing, in research topics from other subjects, or in their interests outside the classroom. You will be able to capitalise better on the students’ initiatives if the arrangements in the classroom are flexible enough to allow individuals or small groups to do extended work at different times from the rest of the class. In this way they can begin while their interest in the topic is fresh.
Cross-Curricular Extended Work

Many pieces of extended work go beyond the boundaries of mathematics. Some have clear links with another school subject and in these cases the student will be pursuing that subject as well as mathematics. The processes of collecting, manipulating and analysing data are mathematical processes, but the data themselves are likely to be of interest to another subject area. Where possible, departments should co-operate, and provide students with the opportunity to pursue their project during the lesson time of relevant school subjects. The list of ideas for Extended Work at the end of this chapter indicates which other school subjects could be involved. This inter-departmental cooperation will be required of us shortly anyway, as there are to be cross-curricular Standard Assessment Tasks for the National Curriculum.

Ideas for Extended Work

Most of the activities in Section Three give suggestions for extensions. Here are some further ideas:

(The other school subject(s) for cross-curricular work are indicated in brackets.)

Maths and Music: (Music, Science)

- The frequency of notes doubles as you go up an octave—investigate
- What is the fret spacing on: —a guitar
  —a violin, etc?
  Give a reason for this.
- How are the lengths of notes represented in manuscript?
- In what ways has manuscript evolved over the centuries?

Maths and Dance/Movement: (Dance/Drama, P.E.)

- Use spatial language to describe movements in dance
- Move in response to mathematical spatial language
- Look at the notation of choreography
- Demonstrate number patterns by movement of people

Maths and Sports: (P.E.)

- Plot the records for one sport against time on a graph. Can you use the graph to make a prediction?
  Plot men’s and women’s records on the same graph

Maths and the Weather: (Science)

- Are pressure and wind speed related?
- Compare climates in different parts of the world
  Interview people who have lived outside the UK

Maths and Cooking: (Home Economics)

- Look at all the different measuring systems used in the different recipe books
  What measures can you find for, e.g. butter oz (ounces), g (grams), tbs (tablespoons), cups
  What measures can you find for, e.g. water oz, g, tbs, cups, pints, fl oz (fluid ounces), ml (millilitres), l (litres)
  How do all these different measures relate to each other?
- Look at oven dials—there are three different systems for measuring how hot the oven is. What are they?
- How do they relate to one another?
Chapter 3: Specific Ways of Working, continued

**Maths and Paperwork:** (Technology, Art)

- Look at different paper sizes: A sizes and B sizes
  - How are they related?
- What about envelope sizes?
- There are different thicknesses of paper
  - What are different thicknesses called?
  - What are they for?
- What quantities is paper sold in?
- What does paper weigh?

**Maths and Gardening:** (Science)

- How can you measure moisture in soil?
- What is the ideal temperature to sprout seeds?
- How fast do seedlings grow?
  - Plot the height against time on a graph
- How much room do plants need?
- How many can you fit in a flower bed?
- Which plants are the best buy?

**Maths and Motoring:** (Science, Technology)

- Grades of petrol, unleaded petrol
- Petrol consumption
- Measure spark-plug gaps using a feeler gauge
  - What are the tolerances for different cars?
- Tyre markings
- Registration plates
- Shapes of various engine parts
- Make a model of the engine

**Maths and Time:**

- Clocks: 12 hour, 24 hour, analogue, digital
- History of months, days of the week
- Different time systems
- The length of a year
- The rotation of the earth